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POPULATION ANALYSIS.—*The extinction of families*—I.[1] ALFRED J. LOTKA, New York.

The theoretical discussion of the chances of extinction of a line of descent was first given by H. W. Watson in 1889, following a suggestion of Francis Galton. Watson showed that the chances of ultimate extinction of a male line of descent, reckoned at the moment of birth of the first male in the line, is given by the positive root inferior to unity, if such a root exists, of the equation for $x$

$$x = C_0 + C_1 x + C_2 x^2 + \ldots \tag{1}$$

where $C_N$ denotes the probability, at birth, that a male shall eventually have just $N$ sons. From the nature of the coefficients $C$ it is clear that

$$\sum_0^\infty C_n = 1 \tag{2}$$

The same problem has also been discussed, essentially with the same results, by J. F. Steffensen (Mathmatisk Tidskrift 1930 p. 19).

The theoretical treatment of the problem is thus available in the existing literature. But there remains to be established a working connection between the analytical formula and available statistical data.

*Probability of eventually having n children.* If $p(a)$ is the probability, at birth, of reaching age $a$, and if $m_n(a)$ is the rate at which $n$th children are born of parents of age $a$ and of given sex, per head of population of age $a$ and of that sex, then evidently, the probability that a newborn child of the given sex shall eventually have *n or more* children is given by

$$\begin{aligned} A_n &= \int_0^\infty p(a)\, m_n(a)\, da \qquad (n = 1, 2, 3, \ldots) \\ A_0 &= 1 \end{aligned} \tag{3}$$

[1] Received August 6, 1931.

and the probability that it will eventually have *just n* children is

$$c_n = A_n - A_{n+1} \tag{4}$$

So far no attention has been paid to the particular sex of the original ancestor and the progeny under consideration. It is now necessary to take this into account, because the numerical values of the characteristics involved are different for the two sexes. Chief interest attaches to the male sex, because the family name is perpetuated in direct ancestral male line. Now the published statistics for the United States do not give directly all the data required to form the products $p\ (a)$ $m_n\ (a)$ for the male sex specifically. It is necessary to obtain them by a series of indirect steps, as follows:

*Data available directly.* The annual reports of births in the United States give explicitly a tabulation of total births[2] in the year of the report, classified *by age of mother and by order of child at birth.*[3]

Another table gives the total births, irrespective of order of child, classified *by age of father and of mother.*[4]

On the assumption, admittedly approximate, that the relation between the ages of father and mother is essentially the same for children of all orders, the data of the first-mentioned table were redistributed, for each order of child taken separately, in accordance with the ratios appearing in the second table, so as to give the number of births classified *by age of father and by order of child at birth.*

Having thus obtained figures for births classified by age of father and by order of child, these figures were then divided each by the corresponding male population of the same age. The result is what may be called the paternity frequency among males of specified age, counting only children of specified order, and *of both sexes*; it is the quantity denoted by the symbol $m_n\ (a)$ in equation (3) above, in the case that we are considering the male parents. We are, therefore, now in a position to determine by (4) numerical values of the quantities $c_n$, which measure the probability that a male just born shall eventually have just $n$ children, counting both sons and daughters.

One step now remains to be performed. From the series of values $c_n$ we must derive the corresponding probabilities $C_N$ that a male just born shall eventually have just $N$ *sons.*

The process of deriving the values $C_N$ from the set of $c_n$'s is best explained by reference to the accompanying Table 1. This is built up from the data appearing in the bottom line of the table, which are the

[2] In the Birth Registration Area. For 1920 see Ann. Rep. p. 201, Table 7.

[3] Counting all the liveborn children of the family, whether living or deceased.

[4] For 1920 see Ann. Rep. p. 193, Table 6.

values of the $c_n$'s (probability, at birth of eventually having just $n$ children), determined according to (4). Each figure appearing in this bottom line has been distributed over the entries in the column above, in accordance with the frequency with which $N$ sons will occur among $n$ children in the same family. This frequency is given by the coefficient of $p$ in $(px + q)^n$, where $p$ and $q$ are, respectively, the probabilities of a birth being male or female. The exact numerical values of $p$ and $q$ for the sex ratio at birth in the general population are 0.515 and 0.485. It is not strictly permissible to use this same figure as representing the sex ratio at birth among children of one family, because within one

TABLE 1.—CHANCES, PER 10,000, THAT A NEWBORN MALE WILL EVENTUALLY HAVE $n$ CHILDREN, COMPRISING $N$ SONS

United States White Males, 1920

| $N$ | $n$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | Σ (2) |
| 0 | 3686 | 787 | 319 | 122 | 44 | 15 | 5 | 2 | 1 | | | | | | | | | | | 4981 |
| 1 | | 788 | 639 | 364 | 177 | 77 | 33 | 14 | 7 | 3 | 1 | | | | | | | | | 2103 |
| 2 | | | 319 | 364 | 266 | 153 | 82 | 43 | 24 | 10 | 6 | 2 | 1 | | | | | | | 1270 |
| 3 | | | | 122 | 177 | 153 | 109 | 73 | 47 | 24 | 15 | 6 | 3 | 1 | | | | | | 730 |
| 4 | | | | | 44 | 77 | 82 | 72 | 58 | 37 | 26 | 12 | 6 | 3 | 1 | | | | | 418 |
| 5 | | | | | | 15 | 33 | 43 | 47 | 36 | 30 | 18 | 11 | 5 | 2 | 1 | | | | 241 |
| 6 | | | | | | | 5 | 14 | 24 | 24 | 26 | 17 | 12 | 5 | 3 | 1 | 1 | | | 132 |
| 7 | | | | | | | | 2 | 7 | 10 | 15 | 12 | 10 | 6 | 4 | 2 | 1 | | | 69 |
| 8 | | | | | | | | | 1 | 3 | 6 | 6 | 6 | 5 | 4 | 2 | 1 | 1 | | 35 |
| 9 | | | | | | | | | | | 1 | 2 | 3 | 3 | 2 | 1 | 1 | 1 | 1 | 15 |
| 10 | | | | | | | | | | | | | 1 | 1 | 1 | 1 | 1 | | | 5 |
| Σ(1) | 3686 | 1575 | 1277 | 972 | 708 | 490 | 349 | 263 | 216 | 147 | 126 | 75 | 53 | 29 | 17 | 8 | 5 | 2 | 1 | 10,000 |

Σ (1): Chance that a newborn male shall have $n$ children.
Σ (2): Chance that a newborn male shall have $N$ sons.

family the sexes of the several children cannot be regarded as independent or "uncorrelated." There is a marked tendency, due undoubtedly to physiological causes, for some families to exhibit a marked preponderance of one sex, either male or female. But as a first approximation we may employ the values of $p$ and $q$ as observed in the general population, and, furthermore, it will greatly simplify our arithmetic if in place of the exact values 0.515 and 0.485 we employ the approximate values 0.5 and 0.5; that is, if we disregard the slight bias in favor of the birth of males.[5]

[5] In the United States the average of the sex ratio at birth over many years is 106 boys to 100 girls.

After the body of the table has thus been filled in by distributing the figures from the bottom line over the corresponding columns, the figures in the last column are finally obtained by summing the figures in each horizontal line. The figures thus obtained, and shown in the last column of Table 1, are the values of $C_N$ that is, the probability for a male, at birth, that he shall eventually have just $N$ sons.

We are now in a position to make numerical application of the fundamental formula (1), in which we have

$$C_o = 1 - A_1$$
$$C_N = A_N - A_N + 1 \tag{5}$$

so that

$$x = 1 - A_1 + (A_1 - A_2)\, x + (A_2 - A_3)\, x^2 + \ldots \tag{6}$$

$$1 = A_1 + A_2 x + A_3 x^2 + \ldots \tag{7}$$

Under ordinary conditions, if a root of (7) inferior to unity exists, it will be but little below unity. The solution of that equation is therefore best effected by introducing a new variable

$$\xi = 1 - x \tag{8}$$

Making this substitution in (7) we obtain, after some simplification,

$$1 = R_o - Q_1 \xi + Q_2 \xi^2 - Q_3 \xi^3 + \ldots \tag{9}$$

where $R_o$ is the ratio of total male births in two consecutive generations, namely

$$R_o = \int_0^\infty p(a) \sum_1^\infty m_N(a)\, da \tag{10}$$

and where the coefficients $Q$ take the form

$$Q_k = \sum_{s=k+1}^{s=\infty} A_s \binom{s-1}{k} \tag{1z}$$

The equation (9) is readily solved by successive approximations, since $\xi$ is small. As applied to the data for the United States in 1920 it gave the following values for $x$, the probability of the male line of descent, from a newborn male, ultimately becoming extinct: First approximation, .8930; second approximation, .8800; fifth and six approximations, identical to four places of decimals, .8797.

PHYSICAL CHEMISTRY.—*The cubic compressibility of certain substances.*[1] L. H. ADAMS and R. E. GIBSON. Geophysical Laboratory, Carnegie Institution of Washington.

In the course of several years we have accumulated results on the cubic compressibility of a number of unrelated solids and liquids. These compressibilities were determined directly at pressures ranging from 1 to 12,000 bars[2] by the piston-displacement method which has been used in the Geophysical Laboratory for the last fifteen years and which is described in detail elsewhere.[3] The work leading to the estimation of the compressibility of a substance by this method may be divided into three main parts.

1. The volume of the specimen, $V_a$, is accurately determined at atmospheric pressure. This gives, incidentally, a reliable estimate of the density or the specific volume of the substance.

2. The accumulation of data necessary for the compilation of a table of $(k - k')$ at intervals of 1 kilobar from 2 to 12 kilobars.[4]

The significance of $k$ and $k'$ may be elucidated by brief reference to the actual experimental procedure. The specimen under investigation is placed in a heavy-walled cylinder or bomb and immersed in *n*-butyl ether. The ether is then compressed between the walls of the bomb and the lower surface of a leak-proof piston which is forced into the bomb. The pressure is measured with an electrical pressure gauge and at even kilobars the displacement of the piston is read. From these readings, after the application of a few corrections, the change in volume of the contents of the bomb for each kilobar rise in pressure may be calculated. In order to eliminate the volume change due to the pressure fluid (butyl ether) and to the distortion in the bomb, a run is made under exactly similar conditions and in identically the same way with a standard material, Bessemer steel, whose compressibility is known. From these two series of readings it is possible to compute the relative change in volume per cm³ of the specimen when the pressure is raised from atmospheric pressure to any pressure $p$.

This relative volume change for which the name, the *bulk compression*,[5] or simply the *compression*, is advocated, is denoted by the symbol $k$.

[1] Received August 5, 1931.

[2] 1 bar = $10^6$ dynes per cm² or a c.g.s. atmosphere.

[3] Adams, Williamson, and Johnston. Journ. Am. Chem. Soc. **41**: *12*. 1919; Adams and Williamson. Journ. Franklin Inst. **195**: *475*. 1923.

[4] 1 kilobar = 1000 bars = 1000 c.g.s. atmospheres.

[5] See Parsons and Cook. Proc. Roy. Soc. **85**: *332–48*. 1911.

Hence $k = \frac{V_a - V_p}{V_a}$ where $V_p$ is volume of specimen at pressure $p$ and $V_a$ is its volume at atmospheric pressure.

For convenience in working we make measurements from an arbitrary zero of pressure, viz. 2 kilobars. The symbol $k'$ is used to denote shrinkage in volume per cm³ produced when the pressure surrounding the substance is increased from 1 to 2000 bars. Hence $(k - k')$ is the decrease in volume which one cm³ of the substance measured at atmospheric pressure suffers when its pressure is raised from 2 kilobars to any pressure $p$.

3. By the method of least squares $(k - k')$ is expressed as a function of the pressure and hence the compressibility $\beta = \frac{d(k - k')}{dp}$ is evaluated.

In this communication we shall classify the results according to the substance examined, giving under each heading the object of the determination, the density of the material, tables of $(k - k')$ at different pressures, and the value of $\beta$ which we consider most probable. The tables of results give $k$ or $(k - k')$ as a function of pressure. In most cases an equation of the form

$$(k - k') = a + bp + cp^2$$

was fitted to the data by the method of least squares. The column labelled "least square residuals" gives the difference between the observed values of $(k - k')$ and those calculated from the appropriate quadratic equation whose coefficients $a$, $b$ and $c$ appear at the head of the table. This column illustrates how well the equation represents the data.

*Standard of comparison.* The cylinder of Bessemer steel which has always been used for this purpose in this Laboratory served as the standard of comparison and Bridgman's[6] value for the *bulk compression* of steel, viz. $10^5 k = 59.9\, p - 0.22\, p^2$, was assumed as a basis on which to compute the results.

*Pyrex glass.* Since piezometers and capsules made from pyrex glass are continually used in this Laboratory, it is desirable for practical reasons to have an accurate knowledge of the compressibility of this material. From measurements of the linear compressibility of pyrex glass, Bridgman[7] gave the following value for the cubic compressibility:

$$10^5 \beta = 304.1 + 1.46\, p \text{ reciprocal kilobars.}$$

A startling feature of this result was the increase in compressibility

[6] P. W. Bridgman. Proc. Am. Acad. Arts Sci. **58**: *174*. 1923.
[7] P. W. Bridgman. Am. Journ. Sci. **10**: *363*. 1925.

with pressure exhibited by this glass. It was considered worth while checking this result by a direct determination of the *cubic* compressibility.

The sample of pyrex glass examined was in the form of a rod $\frac{5}{8}$ inches in diameter and 4 inches long which was practically free from bubbles. It was made available for our use through the courtesy of the Corning Glass Works. The density of the glass determined at 23.2° by the Archimedes method was 2.233 g/cm³ and its refractive index,[8] $n_D$, was 1.473.

The results of the two latest determinations of the compressibility of pyrex glass are given in Table 1.

TABLE 1. EXPERIMENTAL RESULTS FOR PYREX GLASS

| Pressure (kilobars) | 1<br>$a = -615$<br>$b = 302.4$<br>$c = 0.87$ | | 2<br>$a = -620$<br>$b = 306.5$<br>$c = 0.55$ | |
|---|---|---|---|---|
| | $(k-k') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) | $(k-k') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) |
| 12 | 3135 | −4 | 3128 | −9 |
| 11 | 2828 | 12 | 2829 | 11 |
| 10 | 2497 | 1 | 2504 | 4 |
| 9 | 2158 | −19 | 2169 | −14 |
| 8 | 1865 | 6 | 1876 | 9 |
| 7 | 1546 | 1 | 1550 | −3 |
| 6 | 1243 | 12 | 1244 | 5 |
| 5 | 913 | −6 | 917 | −9 |
| 4 | 612 | 4 | 615 | 1 |
| 3 | 287 | −13 | 299 | −6 |
| 2 | 0 | 7 | 0 | 5 |
| 1 | −320* | | −325* | |

* Not used in least square calculation.

The mean of the two results gives the following value of $k$, the total *bulk compression* produced by increase of pressure from atmospheric pressure to $p$ kilobars:

$$10^5 k = 304.5\, p + 0.71\, p^2 \qquad (1)$$

This value is substantiated by two determinations made in 1925 which gave as an average:

$$10^5 k = 303.2\, p + 0.18\, p^2 \qquad (2)$$

Taking into account the recent improvements in our technique we are inclined at this time to give all the weight to equation (1) and to

[8] We are indebted to Dr. H. E. Merwin of this Laboratory for this value.

give as our estimate of the most probable value of the cubic compressibility of pyrex,

$$10^6 \beta = 304.5 + 1.42\, p \text{ recip. kb. at } 25°\text{C.}$$

The agreement between this value and Bridgman's is obvious, and in particular all our results confirm his observation that the compressibility of pyrex increases with pressure.

*Vitreous silica.* According to Bridgman[7], who gives as the compressibility of vitreous silica,

$$10^6 \beta = 269.9 + 3.66\, p \text{ recip. kb at } 30°,$$

TABLE 2. EXPERIMENTAL RESULTS FOR VITREOUS SILICA

| Pressure (kilobars) | 1<br>$a = -560$<br>$b = 273.6$<br>$c = 1.84$ | | 2<br>$a = -563$<br>$b = 275.8$<br>$c = 1.60$ | |
|---|---|---|---|---|
| | $(k-k') \times 10^6$ (obs) | Least square residuals $\times 10^6$ (obs-calc) | $(k-k') \times 10^6$ (obs) | Least square residuals $\times 10^6$ (obs-calc) |
| 12 | 2977 | −11 | 2972 | −4 |
| 11 | 2683 | 11 | 2665 | 1 |
| 10 | 2369 | 9 | 2356 | 1 |
| 9 | 2043 | −9 | 2048 | 0 |
| 8 | 1748 | 2 | 1751 | 5 |
| 7 | 1445 | 0 | 1450 | 4 |
| 6 | 1157 | 9 | 1158 | 9 |
| 5 | 842 | −12 | 837 | −19 |
| 4 | 561 | −3 | 557 | −9 |
| 3 | 277 | 0 | 284 | 5 |
| 2 | 0 | 5 | 0 | 5 |
| 1 | −271* | | −286* | |

* Not used in least square calculation.

this glass exhibits the greatest increase of compressibility with pressure of any substance yet examined. Our results confirm this statement. We made experiments on a cylinder of perfectly clear vitreous silica whose only flaws were a few long, exceedingly fine capillaries—probably elongated bubbles. The volume of the specimen at atmospheric pressure was 20.262 cm³, and its density, determined at 23°C. by the Archimedes method, was 2.204 g/cm³. The specimen came from the General Electric Company's works at Lynn, Massachusetts. Five series of measurements on this specimen were made, three in 1925, and two in 1930. Table 2 gives the results of the last two determinations. On averaging the coefficients in this table we obtain equation (3):

$$10^5 k = 274.7 p + 1.72 p^2 \quad (3)$$

$$10^5 k = 257.4 p + 2.79 p^2 \quad (4)$$

In the three earlier results the values of the coefficients $b$ and $c$ were 258.1, 259.9, and 253.3; and 2.61, 2.86, and 2.90 respectively. The average of these gives equation (4). In assigning a most probable value to the compressibility we have again taken the improvement in our technique into account and have given the later result twice the weight of the earlier one. Our estimate of the compressibility of vitreous silica is, therefore,

$$10^5 \beta = 268.9 + 4.15 p,$$

which agrees very closely with Bridgman's value.

TABLE 3. EXPERIMENTAL RESULTS FOR OBSIDIAN

| Pressure (kilobars) | 1<br>$a = -571$<br>$b = 285.7$<br>$c = -0.58$ | | 2<br>$a = -570$<br>$b = 283.9$<br>$c = -0.38$ | |
|---|---|---|---|---|
| | $(k-k') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) | $(k-k') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) |
| 12 | 2770 | −4 | 2781 | −1 |
| 11 | 2509 | 7 | 2507 | 0 |
| 10 | 2226 | −2 | 2226 | −5 |
| 9 | 1953 | 0 | 1952 | −2 |
| 8 | 1681 | 3 | 1687 | 10 |
| 7 | 1406 | 5 | 1406 | 7 |
| 6 | 1131 | 9 | 1124 | 4 |
| 5 | 824 | −19 | 823 | −17 |
| 4 | 557 | −6 | 550 | −9 |
| 3 | 287 | 6 | 284 | 6 |
| 2 | 0 | 2 | 0 | 4 |
| 1 | −313* | | −287* | |

* Not used in least square calculation.

*Obsidian.* The compressibility of the specimen of obsidian described in a previous paper[9] was redetermined largely with a view to establishing more firmly the sign and magnitude of the pressure coefficient of the compressibility. Glasses with high silica content, e.g. pyrex glass, resemble vitreous silica in exhibiting an increase of compressibility with pressure, while glasses with low silica content behave in the ordinary way. Obsidian is a naturally occurring glass containing over 70 per cent of silica, and its compressibility shows very little

[9] Adams and Williamson. Journ. Franklin Inst. **195**: *483*. 1923.

change with pressure. It is evidently on the border line between the glasses which behave normally under pressure and those which do not. The volume of the specimen we used was 20.007 cm³ and its density at 25°C. was 2.333 grams per cm.³ The experimental results are given in Table 3.

The equation, $10^5 k = 284.8 p - 0.48 p^2$, represents the mean of these results and leads to the following equation for the compressibility:

$$10^5 \beta = 284.8 - 0.96 p.$$

Adams and Williamson found that as far as their results could show the bulk compression of obsidian was a linear function of the pressure and that the value of $\beta$ at 7 kilobars was $2.83 \times 10^{-3}$ reciprocal kilo-

TABLE 4. EXPERIMENTAL RESULTS FOR DURALUMIN

| Pressure (kilobars) | $a = -259$<br>$b = 121.0$ | |
|---|---|---|
| | $(k-k') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) |
| 12 | 1184 | −9 |
| 11 | 1094 | 22 |
| 10 | 947 | −4 |
| 9 | 817 | −13 |
| 8 | 706 | −3 |
| 7 | 592 | 4 |
| 6 | 476 | 9 |
| 5 | 352 | 6 |
| 4 | 212 | −13 |
| 3 | 87 | −17 |
| 2 | 0 | 17 |

bars. This value may be compared with $2.78 \times 10^{-3}$ reciprocal kilobars, which is the compressibility of obsidian at 7 kilobars as computed from the results in this paper. Bridgman[10] measured the compressibility of a sample of obsidian from Ascension Island by the linear method. He interpreted his results as indicating that the compressibility of this natural glass first decreases and then increases as the pressure is raised. His value of the compressibility at 7 kb is considerably lower than ours, being of the order of $2.55 \times 10^{-3}$ reciprocal kilobars. Two considerations prevent close comparison between Bridgman's results and ours. First, the sample of obsidian he examined contained three per cent less silica than our sample—a circumstance

[10] P. W. Bridgman. Am. Journ. Sci. **10**: *384*. 1925.

which would call for a lower compressibility in Bridgman's sample. Second, as Bridgman himself implies, especially in the case of a basaltic glass from Kilauea, the linear method cannot be trusted too far to give the cubic compressibility of natural glasses.

*Duralumin.* The sample of duralumin examined was in the form of a cylinder built of disks and held together by gold wire. Dr. L. B. Tuckerman of the U. S. Bureau of Standards was kind enough to supply the specimen, which was described as 45-duralumin, Bureau of Standards Number 6532 E-4.

Only one determination was made, and the results are given in Table 4. A linear equation was used to express $(k - k')$ as a function of the pressure.

TABLE 5. EXPERIMENTAL RESULTS FOR THREE SALTS

| Pressure (kilobars) | Ammonium nitrate $a = -638$ $b = 668.9$ $c = -13.86$ | | Potassium sulphate $a = -318$ $b = 331.8$ $c = -3.83$ | | Sodium sulphate $a = -212$ $b = 237.1$ $c = -2.38$ | |
|---|---|---|---|---|---|---|
| | $(k-k'') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) | $(k-k'') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) | $(k-k'') \times 10^5$ (obs) | Least square residuals $\times 10^5$ (obs-calc) |
| 10 | 4666 | 1 | 2618 | 1 | 1912 | 1 |
| 9 | 4278 | 19 | 2358 | 0 | 1704 | −16 |
| 8 | 3804 | −22 | 2082 | −9 | 1499 | −25 |
| 7 | 3360 | −5 | 1825 | 8 | 1334 | 10 |
| 6 | 2869 | −7 | 1525 | −10 | 1123 | 4 |
| 5 | 2367 | 7 | 1244 | −1 | 904 | −5 |
| 4 | 1816 | 0 | 948 | 0 | 702 | 8 |
| 3 | 1258 | 14 | 665 | 22 | 490 | 15 |
| 2 | 656 | 12 | 332 | 2 | 263 | 12 |
| 1 | 0 | −17 | 0 | −10 | 0 | −22 |

The average compressibility of this substance between 2 and 12 kilobars is $1.21 \times 10^{-3}$ reciprocal kilobars.

*Solid salts.* We have also determined the compressibilities of three solid salts, *ammonium nitrate, potassium sulphate,* and *sodium sulphate.* The last-named salt was in the form, $Na_2SO_4V$, or thenardite. All these salts received the same treatment. The powdered salt (J. T. Baker's C. P. Analyzed) was pressed into extremely compact pellets with the help of a testing machine capable of exerting a force of 5000 kg. Enough pellets were weighed out to give a total volume of salt of 10 cm³. This was the volume of the salt, not that of the salt plus the pore space. The pellets were placed in a steel capsule and submitted to hydrostatic pressure. A comparison run with 10 cm³ of steel instead of the salt in the capsule enabled us to estimate directly the

difference between the compressibility of the salt and of the standard, steel. The ammonium nitrate was not compressed to pellets but melted and cast into a cylinder. Two series of measurements were made on ammonium nitrate, two on potassium sulphate, and one on sodium sulphate. The average observed values of the *compressions* are recorded in Table 5. Equations (5), (6), and (7) represent the average results for ammonium nitrate, potassium sulphate, and sodium sulphate respectively:

$$10^5 k = 668.9\ p - 13.86\ p^2 \quad (5)$$

$$10^5 k = 331.8\ p - 3.83\ p^2 \quad (6)$$

$$10^5 k = 237.1\ p - 2.38\ p^2 \quad (7)$$

The compressibility, $\beta$, of any of these salts may be obtained by differentiation of the appropriate equation.

*N-butyl ether.* Contrary to expectation, this remarkable liquid, although relatively non-volatile, does not freeze at pressures as high as 12,500 bars at room temperature. In addition it appears to be comparatively fluid at these high pressures and so is an excellent pressure-transmitting liquid. The n-butyl ether we used came from the Eastman Kodak Laboratories. It is listed as No. 173 B.p. 142–144°. Two series of determinations were made over the high-pressure range and a special piezometer method was employed to measure the *bulk compression* up to 2 kilobars, which, it should be remarked, is more than one-third of the bulk compression produced when the pressure is raised from zero up to 12 kilobars.

In Table 6 we record the second series of results. It was found that the points from 7 to 12 kilobars for both series were represented accurately by equation (8),

$$10^4 k = 1058 + 187.8\ p. - 4.88\ p^2 \quad (8)$$

Below 7 kb, however, the differences between the values of $k$ as calculated by equation (8) and those observed increased enormously.

For the second series of results these differences could be represented by the equation

$$10^4 \Delta = 1028 e^{-0.70p} \quad (9)$$

where $\Delta$ is the difference between the observed value of $k$ and that calculated by equation (8). Hence the equation

$$10^4 k = 1058 + 187.8\ p - 4.88\ p^2 - 1028\ e^{-0.70p} \quad (10)$$

was used to express the second set of results and gives the values of $10^4 k$ shown in column 3. The differences at low pressures were greater than might be desired, a fact which is partly due to the difficulty we encountered in estimating exactly the values of $k$ between 0 and 2 kb. In the first series of results where the uncertainties at low pressures were greater, the value of $(k_{obs} - k_{calc})$ at 0, 1, 2, 3, and 4 kb were −30, 42, −4, −15, and 6, respectively, although, as has been mentioned, equation (13) fitted accurately above 6 kb.

It is of interest to note that a mass of *n*-butyl ether which occupies 100 cm³ at atmospheric pressure occupies 89.5 cm³ at 2000 bars and 74 cm³ at 12,000 bars.

TABLE 6. RESULTS FOR N-BUTYL ETHER

| Pressure (kilobars) | $10^4$k (obs) | $10^4$k (calc) | Difference × $10^4$ (obs-calc) |
|---|---|---|---|
| 12 | 2608 | 2609 | −1 |
| 11 | 2532 | 2534 | −2 |
| 10 | 2448 | 2447 | 1 |
| 9 | 2353 | 2351 | 2 |
| 8 | 2246 | 2244 | 2 |
| 7 | 2132 | 2125 | 7 |
| 6 | 1998 | 1994 | 4 |
| 5 | 1843 | 1844 | −1 |
| 4 | 1665 | 1668 | −3 |
| 3 | 1441 | 1452 | −11 |
| 2 | 1156 | 1160 | −4 |
| 1 | 747 | 733 | 14 |
| 0 | 0 | 30 | −30 |

*Conclusion.* This communication illustrates the variety of ways in which the compressibility of matter changes with pressure. At one extreme there are the silica-rich glasses whose compressibility increases markedly at the high pressures. At the other extreme a liquid such as butyl ether decreases in compressibility very rapidly up to five kilobars while at higher pressures its compressibility changes no more than that of a solid like ammonium nitrate. Intermediate between these two extremes come substances like obsidian and duralumin where the decrease of compressibility with pressure is barely detectable over a range of twelve thousand atmospheres. Lastly, the salts may be taken as typical of the ordinary not too incompressible crystalline solid whose rate of decrease of compressibility, although large at first, diminishes steadily as the pressure rises.

It will, therefore, be obvious that any general rules for estimating the magnitude of the change of compressibility of a substance from observations on other substances should only be applied after careful consideration of the nature and structure of the substances involved. In particular, rules deduced for the rate of change of compressibility of holocrystalline materials should not in general be applied to glasses.

PALEONTOLOGY.—*New Carboniferous invertebrates*—III.[1] GEORGE H. GIRTY, U. S. Geological Survey.

**Schizophoria peculiaris** n. sp.

Figs. 1–4

Shell very small, broadly ovate in outline, varying in proportions but always wider than long, highly convex, coarsely striated.

The pedicle valve may be compared to a cone which is elliptical at the base, very depressed, very oblique, and considerably arched in the apical part. The hinge line is equal to about half the greatest width but when the valve is viewed from above it is mostly concealed by the projecting umbonal parts. The outline, due to these two factors, has an ovate shape but little interrupted by the cardinal angles, with well rounded and strongly divergent sides and a broad gently convex or sometimes faintly emarginate anterior margin. The cardinal area is high, but not so high as it is wide and it is neither strongly arched nor strongly inclined backward. The beak tapers rapidly to a point and is not greatly incurved. The convexity is high and regular, though chiefly localized in the posterior part and the median line may be depressed into an obscure sinus.

The brachial valve differs in shape but little from the pedicle valve. The cardinal area is lower and it lies essentially in the plane of the shell margins. Though lower than the cardinal area of the pedicle valve, it extends almost as far beyond the hinge line and is relatively high for a shell of this kind and size. The beak (compared with the ventral beak) is blunter and more incurved. A median sinus is of more common occurrence in this valve than in the other, and though never strong, it is likewise more distinct. If a sinus is present, and especially if a sinus is also present in the pedicle valve, the outline in front is correspondingly emarginate.

The surface is marked by radial lirae that are exceptionally strong and coarse for a shell of this size. They are separated by relatively wide interspaces, though many of the interspaces are partly occupied by interstitial lirae which subsequently become as large as the others. The lirae vary in size and spacing and the surface generally has an irregular appearance. To this appearance of irregularity the tendency of the larger lirae to end abruptly in a large pore or hollow spine contributes somewhat, and also the varices of growth that some specimens developed. Hollow, prominent and interrupted lirae are not a conspicuous feature and on many specimens they are difficult to distinguish at all. They are mostly to be looked for on the pedicle valve. The varices of growth which are especially common and especially strong on these shells, of course interrupt all the lirae simultaneously.

[1] Published by permission of the Acting Director of the U. S. Geological Survey. For the previous papers of this series, see this JOURNAL 19: 135–142 and 406–415. Received July 29, 1931.

Most of the Schizophorias in our American Carboniferous faunas are large shells. The present species differs from them not only in size, but in other respects, so much that detailed comparisons seem unnecessary. That the shells included here merely represent the immature condition of some larger species seems unlikely, not only because they are associated with no large species of which on other grounds they could be the immature form, but because they occur in considerable numbers and are more or less uniform in size and in other characters. The largest specimen referred here has a width of 12 mm., the next largest, 10 mm. and so on down.

Horizon and locality: Upper part of the Hueco limestone; Marble Canyon, east face of the Diablo Plateau, Hudspeth County, Texas (station 6681).

**Pustula palmeri** n. sp.

Figs. 5–10.

Shell small, rarely 10 mm. in width, highly convex (in the pedicle valve), lacking both radial costae and concentric wrinkles that are either systematic or pronounced but marked by numerous small spines which project from large rounded bases.

Pedicle valve subquadrate in outline except for the projecting umbonal parts, the front and sides being more or less straightened merging in strong curves around the antero-lateral angles. The proportions vary in different specimens, but the width seems to be regularly greater than the length. The hinge line is about equal to the width below, but it is more apt to be slightly shorter than slightly longer. The shell is strongly and rather regularly arched. Transversely the curvature is somewhat subdued across the median part with a steep descent on either side to the small depressed auricles. The umbonal parts are rather prominent and they project considerably beyond the hinge when the valve is viewed from above but not more than is common.

The surface is primarily marked by spines which arise from large well defined nodes. The spines are numerous and though actually small, are rather large for the size of the shell. They are more or less regularly arranged on a plan which is fundamentally alternating or quincunxial. This arrangement, however, is not as a rule conspicuous, a formation in transverse rows being more obvious than any other. In some specimens curving oblique rows are formed at the sides and in others the development of spines and nodes is accompanied by a tendency to form large indistinct radial costae on the anterior parts. The nodes and spines become gradually smaller and more closely arranged toward the hinge line, near which all the transverse features converge to some extent. Several specimens have a row of small spines along the hinge margin, but this may not be a constant feature. On one of these especially the spines were developed in connection with a series of small, sharp, regular plications extending downward from the margin, a spine to each plication. The spines with their nodose bases are more numerous on some specimens than on others, and aside from number, the nodes vary also in size and prominence. On some specimens they are by no means conspicuous. The surface is marked by striae of growth in the nature of microscopic and irregular concentric wrinkles. Some of these are delicately lamellose. Though not developed regularly or at regular intervals, other striae much larger than the incremental ones occur here and there between the rows of

spines and pustules. More regularly large wrinkles or plications are developed near the hinge line but they lose their strength within a short distance except as they may be represented by the stronger grooves that pass between and emphasize the rows of spines.

The brachial valve corresponds in shape with the pedicle valve in the usual manner. It is strongly transverse and in outline rather subquadrate than semi-circular. It is gently concave so that a relatively ample cavity is left within. The curvature is not quite regular, being low over the visceral disk and more pronounced over the marginal parts without, however, any marked discrepancy.

The most conspicuous surface feature in this valve consists of small rounded indentations or dimples which evidently correspond individually to the nodes on the surface of the pedicle valve. These indentations become deeper and more elongate toward the cardinal margin, so that the elevations between them take the shape of more or less irregular wrinkles. Microscopic wrinkles of incremental nature cover the entire surface which sometimes also is marked by concentric undulations (broad grooves and narrow ridges), all very faint but fairly regular where they occur at all.

This valve also is armed with spines which are at least as numerous as the spines of the pedicle valve and much smaller. They occur distributed among the dimples and spring from the surface without giving rise to appreciable nodes.

Species of the general character of *P. palmeri* are rare in our Pennsylvanian faunas and I am able to name but two which can profitably be compared with it. *P. keytei* has fewer and smaller spines which were developed without appreciably disturbing the even curvature of the pedicle valve by giving rise to nodes or spine bases. Although the nodose or monticulate surface is not a constant feature in *P. palmeri*, at least in a conspicuous degree, it is a very common one. *P. keytei* is also somewhat smaller and it is somewhat differently proportioned, being comparatively long whereas *P. palmeri* is comparatively broad. *Pustula globosa* is more comparable to *P. palmeri* in size and shape but it has much fewer spines and the spines, though leaving large scars on the surface, did not produce distinct elevations in the nature of spine bases.

Horizon and locality: Cherokee shale; mine dumps near Joplin, Missouri.

**Pustula keytei** n. sp.

Figs. 11–14.

Shell very small, hemispherical, without radial costae, and without concentric plications, but covered with numerous small spines.

Pedicle valve irregularly ovate in outline with length and width about equal though commonly somewhat longer than wide. Outline in front of the hinge rather regularly rounded. Hinge a little shorter than the width in front, mostly concealed by the umbonal parts when the valve is viewed from above. Convexity high. Umbonal region rather gibbous descending abruptly to the small, ill-defined auricles and projecting well beyond the hinge. A section longitudinally would show the greatest height posterior to the mid-length with a gradually decreasing curve forward.

The surface, which is without even traces of radial costae, is marked by fine, incremental lines. The sides of the vault and the auricles show small subequal concentric plications most of which spread out and become indistinguishable over the intermediate parts. A few of the grooves may persist, marking off concentric bands but this is more or less sporadic and developed especially on the visceral disc. Some of the striae also are lamellose along the posterior side. The spines which are represented by scars or by fragments of the basal parts, were apparently small and perpendicular to the surface, from which they were developed without giving rise to conspicuous nodes or spine bases. They show a general tendency to form concentric rows and occur scatteringly over the median part of the vault but in more close arrangement on its sides.

The brachial valve is semicircular in outline. The width, though greater than the length, is by no means twice as great. The curvature is moderately strong but much lower than that of the pedicle valve so that a considerable space is left between them. The curvature is also fairly regular—perhaps a little strengthened over the marginal parts. The surface features are so subdued as to be rather inconspicuous. Very fine incremental striae can be seen and descending from the hinge fine obscure wrinkles which either lose themselves in passing across the intermediate part or give rise to faint concentric undulations. Scattered over the surface, though scarcely to be seen except in a favorable light, are small dimple-like depressions that correspond to spines on the pedicle valve and were apparently developed simultaneously with them. This valve also was provided with spines but in comparison with the spines of the pedicle valve, they are smaller and less numerous. Because they are small, however, some may have been overlooked and their number may really be greater than it appears.

Mr. Mather has described a species very similar to this from the Morrow group of Arkansas, of Pottsville age. I have a number of specimens from the Morrow that would naturally be referred under *P. globosa* and among them it is possible to find individuals that do not differ greatly from *P. keytei*, but most of the specimens and apparently the more typical ones, are relatively broader and were armed with distinctly larger spines. The spines are not only larger but apparently also less numerous on the sides of the vault and less closely arranged; the specimens from Arkansas, however, are not as favorably preserved as those from Colorado, and it has not been possible to compare them satisfactorily in a number of details. The superficial features of the brachial valve, for instance, are neither adequately described by Mather, nor are they in certain details adequately shown by my specimens. This valve is described as flattened over the visceral disk and abruptly deflected around its margins. Here again the two species appear to differ inasmuch as the brachial valve of *P. keytei* is regularly arched. It should be remarked, however, that some of my specimens of *P. globosa* (if correctly identified) are equally regular though others have a flattened visceral disc and a deflected margin as described. Another similar but less closely related form is *P. palmeri*. In *P. palmeri* the spines are more numerous and more

regularly arranged and they spring from relatively large, rounded nodes which, by reason of their regular arrangement, produce here and there the appearance of large, ill-developed radial costae.

Horizon and locality: Fountain formation (Glen Eyrie shale member); Glen Eyrie, El Paso County, Colorado (station 5100).

**Avonia williamsana** n. sp.

Figs. 15–19

Pedicle valve small, rarely more than 15 mm. in width. Width and length about equal, though the width is commonly somewhat the greater. Outline broadly subovate. Vault moderate in height, gradually enlarging, conspicuously ovate and without a sinus. Hinge line equal to the width in front. Auricles quadrate in outline, small, depressed, ill-defined.

Sculpture comprising growth lines, concentric striae, spines with their bases, and radial costae. The growth lines are fine and inconspicuous. The concentric striae may be fairly strong and numerous, or on the other hand, few and obscure. They are not restricted to the visceral disk but may occur at irregular intervals throughout the entire length of the valve. Not rarely they are so shaped as to make a step-like descent forward. Broadly speaking, the posterior half of the valve is marked by spiniferous nodes and the anterior half by spiniferous costae, but the one type of sculpture graduates into the other, and the change occurs at different stages of growth in different specimens. The umbonal region which is somewhat finely and irregularly corrugated, is marked by numerous nodes which are rather large and surmounted by spines which are rather small. In a general way the nodes occur in transverse rows and alternate in the rows. As growth proceeded the nodes became more elongated, passing into short costae that are interrupted by the transverse striae and these shortly became continuous costae that the weakened striae fail to disturb. The spines are numerous over all parts of the surface and produce nodes where they occur, but where the spines take off from continuous costae, the nodes do not conspicuously deform them. The transverse arrangement of the spines in rows in many places is very striking, but the rows are irregular, crooked, or discontinuous. The costae are strong and rather regular; about 7 or 8 occur in 5 millimeters.

The brachial valve is deeply semicircular in outline and considerably wider than long. In the form of an external mold it is moderately convex but it is decidedly less convex than the pedicle valve, especially over the umbonal parts. These are rather exceptionally prominent for a brachial valve, however, and indeed the curvature of the whole valve though varying in different specimens is exceptionally strong and rather regular.

The surface markings, still in terms of the external mold, are a close repetition of the surface markings of the pedicle valve. Small nodes cover the posterior part and pass by gradual elongation into continuous costae, also faintly nodose, the costae corresponding to the costae of the pedicle valve and the nodes to the spines upon them. This valve equally with the other, is equipped with numerous small spines, and here also the spines are conspicuously but irregularly alined in transverse rows. They occur in the grooves between the costae and the grooves are slightly depressed or enlarged where they occur. Of course the grooves represent costae on the shell itself and

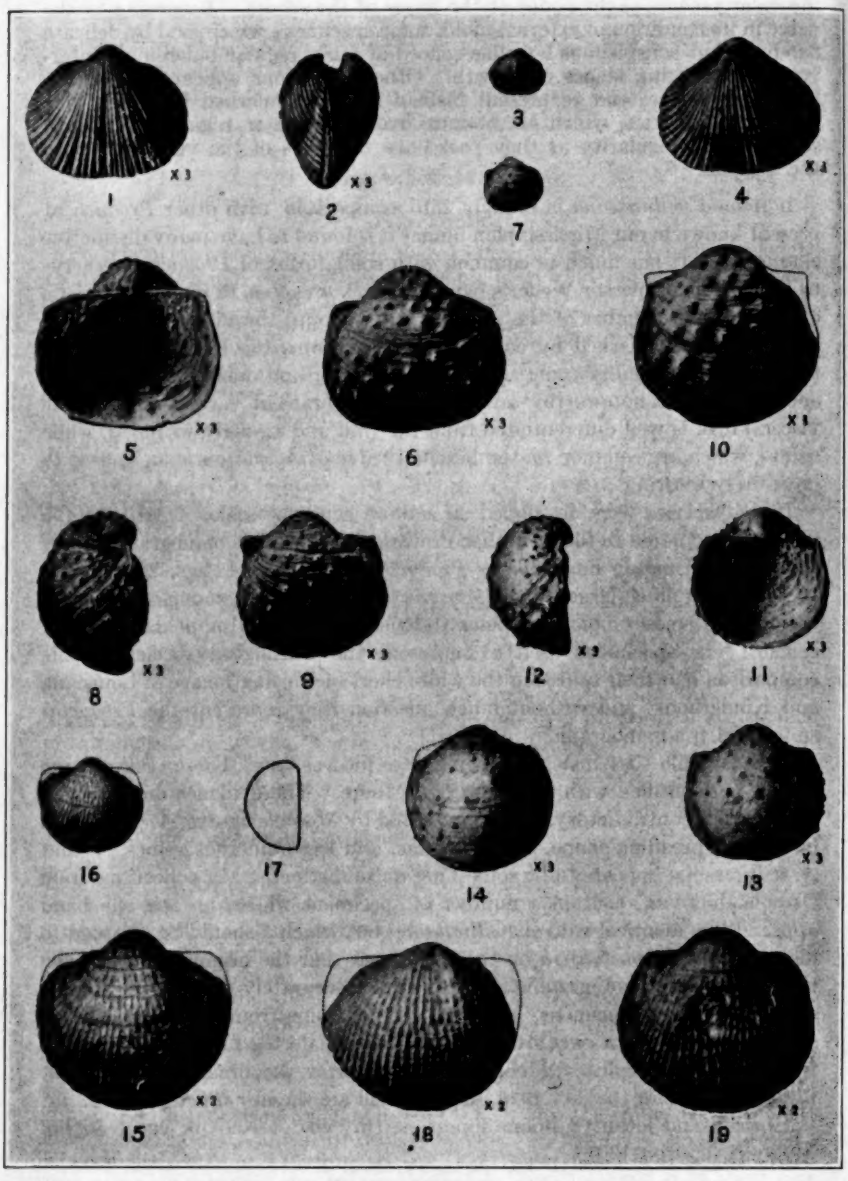
the enlargements slight nodes at the bases of the spines. Returning to the valve in its condition of external mold, some specimens are crossed by delicate flat-lying but conspicuous lamellae spaced at fairly regular but remote intervals and marking stages of growth. Other specimens appear to lack the lamellose feature, and to present instead a surface marked by rather fine concentric wrinkles, which are obscure over the median region but gain in strength and regularity as they pass onto the sides of the vault and onto the auricles.

If *Avonia williamsana* is brought into comparison with other *Producti* at present known in our Mississippian faunas it is found to have many distinctive characters. It has much in common with some forms of *Productus arcuatus* to judge from Professor Weller's figures, but *P. arcuatus*, in its characteristic expression is a member of the *semireticulatus* group; the visceral disc in the pedicle valve is marked by concentric corrugations, the costae are coarser than those of *A. williamsana* and the spines fewer and much larger. Differences still more noteworthy are found in the brachial valve in which the visceral disk is well differentiated from the trail and strongly wrinkled, while spines, which are common on the brachial valve of *A. williamsana*, appear to be entirely wanting.

If that species were developed on a more generous scale, *A. williamsana* might be compared to the form that Professor Weller in his monograph figures from the Burlington limestone as *Productus* sp. (plate 14, figs. 26 and 27). Besides being much larger *A. williamsana* is distinguished among other differences by the more numerous spines and the more gradual appearance of the costae. The specimens used to represent the Burlington species in this comparison had their source in the white chert of Burlington age at Louisiana and Kinderhook, and without much question they represent the *Productus* sp. of Weller's monograph.

It is possible that instead of being described as new, *Avonia williamsana* should be identified with *A. pustulifera* Moore. The evidence on this head is somewhat contradictory. As represented by Moore's figures, *A. pustulifera* has a less spreading shape, coarser costae, and less numerous spines, so that *A. williamsana* appears to be something quite distinct. My collections from Osceola, however, contain a number of specimens which, on the one hand appear to be identical with *A. williamsana* but which I should be tempted to identify with *A. pustulifera* on the hypothesis that the original figures, even though they are photographic, do not fully or accurately portray the specific characters. My specimens, however, did not come from the same horizon as Doctor Moore's, a circumstance that may vitiate the fact that they came from the same locality. Moore did not describe the brachial valve of *A. pustulifera* so that the factors of identification are thereby much reduced.

Horizon and locality: Boone limestone (in beds of Keokuk age); Joplin, Missouri (station 1301A).




Figs. 1–19. For explanation, see page 397

DESCRIPTION OF FIGURES

*Schizophoria peculiaris* n. sp.

Figs. 1–4. Different views of one of three cotypes. Fig. 3 is of the natural size; the others are enlarged to about 3 diameters. Hueco limestone; Marble Canyon, east face of Diablo Plateau, Hudspeth County, Texas, (Station 6681).

*Pustula palmeri* n. sp.

Figs. 5–10. Different views of 3 cotypes, figs. 5–8 representing one specimen, fig. 9 another, and fig. 10 a third. Fig. 7 is of the natural size; the others are enlarged to about 3 diameters. Cherokee shale; mine dumps near Joplin, Mo.

*Pustula keytei* n. sp.

Figs. 11–14. Different views of two cotypes, all ×3. Figs. 11–13 represent one specimen, fig. 14 another. Glen Eyrie shale member of the Fountain formation; Glen Eyrie, El Paso County, Colorado.

*Avonia williamsana* n. sp.

Figs. 15–19. Different views of 3 out of 7 cotypes. Figs. 15–17 represent one specimen, fig. 15 being an enlargement to two diameters. Fig. 18 represents an external mold of a brachial valve ×2. Some of the holes made by the small spines are here shown. Fig. 19 represents a pedicle valve, ×2. This specimen like the original of fig. 15 is practically an internal mold. Cherty beds of the Boone limestone; Joplin, Mo., (Station 1301A).

## ZOOLOGY.—*The copepod genera* Broteas *Lovén,* Paradiaptomus *Sars,* Lovenula *Schmeil,* Metadiaptomus *Methuen, and* Adiaptomus *Cooper.*[1] C. DWIGHT MARSH, Bureau of Animal Industry.

In connection with a study of *Diaptomus* it became necessary to determine somewhat clearly what species should be separated from *Diaptomus* and referred to the proposed genera *Broteas, Paradiaptomus, Lovenula, Metadiaptomus,* and *Adiaptomus.* It was found difficult to get from the authors who have used these names any method of determining by a few characters the generic limits of these proposed divisions of the Diaptomidae. Gurney, 1929, discussed the matter somewhat elaborately, but did not make clear a practical method of separating the genera. This led the writer to go over the literature to see if he could find out how the authors used these names and whether it might be possible, in some fairly simple way, to make diagnoses of the genera. This was necessarily nothing but a study of the literature, as no material of these forms was available for examination. Such a critical study is always difficult because many published descriptions are incomplete and there is always a possibility, especially in the examination of minute structures, that there may be mistakes of observation, in making sketches, or of interpretation. When there is a discrepancy between authors, it may be assumed that the later author is right, although, of course, this would not always be true.

In 1847 Lovén proposed the generic name *Broteas* for a South African copepod giving a somewhat elaborate description with figures. This

[1] Received August 3, 1931.

differed from the recognized characteristics of *Diaptomus* in that the female abdomen was composed of two segments instead of three, the maxillipeds were long and powerful instead of rather weak and inconspicuous, the endopodites of the swimming feet were all two-segmented, while in *Diaptomus* the endopodites of the first feet are two-segmented and of the second, third, and fourth three-segmented, and the first antennae contained 27 segments instead of 25. The exopodite of the left 5th foot of the male was armed with a conspicuous hook, apparently terminal, and an acute spine.

In 1895 Sars described *Paradiaptomus lamellatus*, proposing the new genus *Paradiaptomus*. This, like *Broteas*, has two-segmented female abdomen and long and powerful maxillipeds; the endopodite of the maxilliped consisted of three segments instead of five as in *Diaptomus*. The swimming feet were like *Diaptomus*,—first foot two-segmented and the others of three segments. The exopodite of the left 5th foot of the male was armed with three spines.

In Giesbrecht and Schmeil, 1899, Schmeil proposed *Lovenula* in place of *Broteas*, since *Broteas* was preoccupied. His description was practically like that of Lovén, but he added that the endopodite of the maxilliped consisted of three segments. Thus the main distinction between *Lovenula* Schmeil, or *Broteas* Lovén, and *Paradiaptomus* Sars was that the endopodites of the swimming feet of *Lovenula* have *two segments*, while those in *Paradiaptomus* Sars are like those in *Diaptomus*, two-segmented in the first, and three-segmented in the others. Giesbrecht and Schmeil, 1898, diagnosed *Paradiaptomus* in accordance with the description of Sars as having the two-segmented female abdomen and three-segmented endopodite of the maxilliped as in *Lovenula* and the segmentation of the endopodites of the swimming feet as in *Diaptomus*.

Sars in 1899, from some collections in which he recognized, as he thought, the original species of Lovén, published a detailed description under the title "On the Genus *Broteas* of Lovén with Description of the Type Species: *Broteas falcifer* Lovén:" this description was accompanied with rather elaborate illustrations. He stated that Lovén was doubtless in error in the number of antennal segments which are 25 instead of 27. The last segment of the right antenna bears a small hook. The swimming feet, instead of having two-segmented endopodites, are like those in *Diaptomus*. He considered that these facts make *Broteas* identical with his *Paradiaptomus* and, as *Paradiaptomus* is the more recent name, he made it a synonym of *Broteas*.

Van Douwe, 1912, made some criticisms of preceding descriptions, but stated nothing that adds to our knowledge of the generic distinctions.

Gurney, 1904, in his description of "*Lovenula mea*" which is now considered identical with *Paradiaptomus falcifer* stated that the endopodites of the maxillipeds were *four*-segmented.

Brady, 1913, considered that *Paradiaptomus* should be used to include *Broteas* and *Lovenula*.

Grochmalicki, 1913, made *Paradiaptomus* Sars a synonym of *Lovenula* Schmeil, and listed 3 species of *Lovenula*, *L. falcifera* Loven, *L. lamellata* Sars, and *L. mea* Gurney. He added another species, *L. stolzmanni*.

Rühe, 1921, stated positively that the endopodite of the maxilliped of *Paradiaptomus* has *five* segments. Rühe claimed that Sars was in error in regard to the left fifth foot of the male; he stated that two spines are borne on the external margin of the second segment of the exopodite and a hyaline lamella on the internal margin; that there is a definite line separating the first and second segments which leave the spines on the second segment. Both Sars and Rühe make the left exopodite two-segmented, but they differ in the location of the division.

Sars, 1927, published descriptions of *Lovenula falcifera* and *L. barnardi*, and revived the genus name *Paradiaptomus* for *L. lamellatus*. In both *Lovenula* and *Paradiaptomus* he found the swimming feet like *Diaptomus*, the two-segmented female abdomen, and the endopodites of the maxillipeds *five-segmented*. He made *Paradiaptomus* somewhat different in form from *Lovenula*, the maxillipeds less powerfully developed, and no hook on the ultimate segment of the right male antenna.

Brehm, 1927, stated that the family Diaptomidae has only two genera, *Paradiaptomus* and *Diaptomus*, evidently discarding *Broteas* and *Lovenula*.

In 1906 Cooper proposed the name *Adiaptomus* for a species which he named *A. natalensis*. He stated that it has antennae of 26 segments, female abdomen of 2 segments, and his figure shows the male fifth foot of the same general form as that found in *Lovenula* Schmeil. The endopodites of the maxillipeds are three-segmented, and the swimming feet are like *Diaptomus*. There seems to be no reason for separating *Adiaptomus* from *Lovenula* except the 26-segmented antennae and possibly there was a mistake about this.

*Metadiaptomus* was proposed by Methuen in 1910 with the type species *M. transvallensis*. The antennae are of 26 segments, as stated

for *Adiaptomus*. The female abdomen is three-segmented. The swimming feet are like *Diaptomus*. In the male fifth feet, fig. 3, the second basal segment of the right foot is very much enlarged on the inner border, and this border and part of the dorsal surface is spinose. The left exopodite of the male fifth foot is one- or possibly two-segmented and bears two processes, a strong curved spine on the dorsal surface which, in the type species, is nearly as long as the whole exopodite, and a small terminal spine.

Gurney, 1929, recognized the two genera *Paradiaptomus* and *Metadiaptomus*, dividing *Paradiaptomus* into a *Paradiaptomus* and a *Lovenula* series.

The various ideas about the genera are summarized in Table 1. *Diaptomus* is added for comparison.

It appears that the original descriptions of *Broteas* and *Lovenula* made them differ from *Diaptomus* in the prominence of the maxillipeds, the segmentation of the swimming feet, the female abdomen, and the endopodite of the maxilliped. *Paradiaptomus* Sars 1895 differs from *Broteas* Lovén and *Lovenula* Schmeil in having the swimming feet like those of *Diaptomus*, and in the structure of the exopodite of the left 5th foot of the male. In regard to the segmentation of the endopodite of the maxilliped there has been a progressive change in authors' statements, the more recent making it five, like *Diaptomus*. If we assume, as one naturally would, that the later authors are more correct, it is probable that we should consider that this segmentation is five. It appears, however, that while the division of this endopodite in *Diaptomus* is clearly in five segments they are not so clearly marked in the other genera and hence have been sometimes overlooked. The maxilliped is highly developed and conspicuous in *Paradiaptomus*, *Lovenula*, and *Broteas*, and inconspicuous in *Diaptomus* and *Metadiaptomus*. There is a small hook on the ultimate segment of the right male antenna in *Broteas* and *Lovenula* of Sars and *Paradiaptomus* of Gurney.

The female abdomen in *Broteas*, *Paradiaptomus*, *Lovenula*, and *Adiaptomus* is 2-segmented, while in *Diaptomus* and *Metadiaptomus* it is 3-segmented. This is not a hard and fast distinction, however, for a few species of *Diaptomus* have 2-segmented abdomens. Sars writes of some *Diaptomi* in which the female abdomen has the last two segments "confluent." One might raise the question whether, in such cases, if they were "confluent," it might not be better to consider the abdomen to be two-segmented.

There are differences in the male fifth feet which help materially in distinguishing the genera. As shown in the copy of Sars' figure of the fifth feet of *Broteas falcifer* (Fig. 1), which now should be called *Lovenula*, the left exopodite has a well developed terminal hook and an acute spine. The left endopodite is either lacking or rudimentary.

TABLE 1. SUMMARY OF CHARACTERS

| | Swimming feet | Female Abd. | Endop. Maxilliped | Ult. seg. male rt. ant. | Armature left endop. of male 5th ft. |
|---|---|---|---|---|---|
| Broteas Loven 1847 | 1–4, 2 seg. | 2 seg. | | | Hook & acute spine. |
| Paradiaptomus Sars 1895 | 1, 2 seg.<br>2–4, 3 seg. | 2 seg. | Prominent,<br>Endop. 3 seg. | | 3 stout spines. |
| Lovenula Schmeil 1898 | 1–4, 2 seg. | 2 seg. | Prominent,<br>Endop. 3 seg. | | Hook & acute spine. |
| Paradiaptomus 1898 | 1–2 seg.<br>2–4, 3 seg. | 2 seg. | Prominent,<br>Endop. 3 seg. | | 3 spines. |
| Broteas Sars 1899 | 2–4, 3 seg. | 2 seg. | Prominent,<br>Endop. 3 seg. | Ter. hook. | Hook & acute spine. |
| Lovenula Gurney 1904 | | | Prominent,<br>Endop. 4 seg. | | Hook & acute spine. |
| Paradiaptomus Brady 1913 | | 2 seg. | | | Hook & acute spine. |
| Paradiaptomus Rühe 1921 | | | Prominent,<br>Endop. 5 seg. | | 2 spines & hyaline lam. |
| Lovenula Sars 1927 | 2–4, 3 seg. | 2 seg. | Prominent,<br>Endop. 4 seg. | Ter. hook | Hook & acute spine. |
| Paradiaptomus 1927 | 2–4, 3 seg. | | Prominent,<br>Endop. 4 seg. | | 3 spines. |
| Paradiaptomus, including Lovenula, Gurney 1929 | | 2 seg. | Prominent,<br>Endop. 5 seg. | Ter. hook | |
| Adiaptomus Cooper 1906 | 2–4, 3 seg. | 2 seg. | Endop. 3 seg. | | Hook & spine. |
| Metadiaptomus Methuen 1910 | 2–4, 3 seg. | 3 seg. | | | Large curved spine on posterior surface. Small spine near end. |
| Diaptomus | 2–4, 3 seg. | 3 seg. | Not prominent<br>Endop. 5 seg. | | |

*Paradiaptomus* Sars, as shown in his figure of *P. lamellatus* (Fig. 2) has a distinct left endopodite and the exopodite is armed either with three rather stout spines, as stated by Sars, or with two spines and a hyaline lamella according to Rühe.

*Adiaptomus* has male fifth feet like *Lovenula*. *Metadiaptomus*, as shown in the figure from Methuen of *M. transvaalensis* (Fig. 3), has a stout curved spine on the posterior face of the exopodite and a smaller terminal spine. In addition, the second basal segment of the right foot is much enlarged on the inner side and the inner border and part of the posterior surface is armed with spinules.

The following suggestion then is made for distinguishing between these genera:

*Lovenula* Schmeil has 2-segmented female abdomen, prominent maxillipeds, the left exopodite of the male fifth foot armed with a terminal hook and an acute spine.

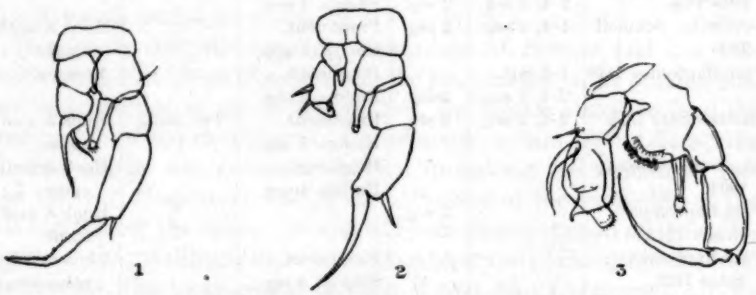



Fig. 1. Fifth foot of male of *Lovenula falcifera*. After Sars.
Fig. 2. Fifth foot of male of *Paradiaptomus lamellatus*. After Sars.
Fig. 3. Fifth foot of male of *Metadiaptomus transvaalensis*. After Methuen.

*Paradiaptomus* Sars has 2-segmented female abdomen, prominent maxillipeds, the left exopodite of the male fifth root armed with 2 stout spines and a hyaline lamella.

*Metadiaptomus* has 3-segmented female abdomen, maxillipeds not prominent, the left exopodite of the male fifth foot armed with a curved spine on the posterior surface and a small terminal spine. The second basipodite of the right male fifth foot is enlarged on the inner side and spinulose.

*Adiaptomus* is a synonym of *Lovenula*.

*Lovenula* will include *Broteas falcifer* Loven 1847, *Lovenula excellens* Kiefer 1921, *Adiaptomus natalensis* Cooper 1906, *Diaptomus bouvieri* Daday 1910, *Paradiaptomus biramata* Lowndes 1930, *P. biramata* Rühe 1921, *P. biramata* Van Douwe 1912, *Broteas falcifer* Sars 1899,

*Lovenula falcifera* Sars 1927, *L. barnardi* Sars 1927, *Paradiaptomus falcifer* Brady 1913, *Lovenula* Giesbrecht & Schmeil 1898, *Broteas falcifer* Metheun 1910, *Lovenula mea* Gurney 1904, *Diaptomus africanus* Daday 1908, *D. aethiopicus* Daday 1908, *Lovenula simplex* Kiefer 1929, *Diaptomus pictus* Brady 1913 (which as suggested by Gurney 1929 is probably a synonym of Cooper's *Adiaptomus natalensis*), and possibly *Diaptomus alluaudi* DeGuerne & Richard 1890 (but female has 3-segmented abdomen).

*Paradiaptomus* will include *Paradiaptomus lamellatus* Sars 1895, *P. lamellatus* Rühe 1921, *P. lamellatus* Giesbrecht & Schmeil 1898, *Broteas* lamellatus Sars 1899, *Paradiaptomus lamellatus* Sars 1927, and *Lovenula stolzmanni* Grochmalicki 1913.

*Metadiaptomus* will include *M. transvaalensis* Methuen 1910, *Paradiaptomus transvaalensis* Kiefer 1929, *Paradiaptomus colonialis* Kiefer 1928, *Diaptomus meridianus* Van Douwe 1912, *D. mascalus* Brady 1913, *D. capensis* Sars 1901, *D. purcelli* Sars 1901, *D. chevreuxi* De Guerne & Richard 1894, *D. alluaudi* De Guerne & Richard 1890, *D. rigidus* Sars 1927, Probably *D. asiaticus* Uljanin 1875, *D. rehmanni* Grock 1913, and *D. tibetanus* Daday 1908.

*Diaptomus greeni* Gurney 1907, called *Paradiaptomus greeni* Gurney 1931, and the two species described by Van Douwe 1912 as *Paradiaptomus schultzei* and *P. similis*, are considered as probably *Diaptomi*. Gurney 1907 stated that the female abdomen in *D. greeni* was 3-segmented. Van Douwe stated that the female abdomen was 2-segmented in *Paradiaptomus schultzei* and *P. similis*. Gurney, 1931, reported as the result of an examination of new material of *P. similis* that the female abdomen is 3-segmented: this would be presumptive evidence that these forms should be considered as *Diaptomi*. It seems probable, moreover, that *P. schultzei* and *P. similis* are not specifically distinct and should be considered as synonymous.

As pointed out by Gurney, 1929, it is sometimes difficult to separate these genera. The typical large, curved spine on the posterior surface of the exopodite of the male fifth foot in *Metadiaptomus* may be much smaller in some species and may be found more distad on the exopodite until it is sometimes difficult to decide whether it should be considered as located on the posterior surface or as terminal. That is, the terminal hook of *Lovenula* is doubtless homologous with the curved spine of *Metadiaptomus*. Probably it is also homologous with the "external" spine of *Paradiaptomus*.

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ZOOLOGY.—Metagonimoides oregonensis, *a new trematode from a raccoon.*[1] EMMETT W. PRICE, Bureau of Animal Industry, United States Department of Agriculture. (Communicated by ELOISE B. CRAM.)

Among some specimens recently referred to the writer by Dr. D. Sinitsin were a few specimens of a trematode which had been collected by Dr. J. N. Shaw, Oregon State College, Corvallis, Oregon, from the intestine of a raccoon. This fluke is closely related to species of the heterophyid genus *Metagonimus* Katsurada, but differs from them in certain characters which are regarded as sufficient to warrant the erection of a new genus. For this form the name *Metagonimoides oregonensis*. n.g., n. sp., is proposed.

**Metagonimoides** new genus.

*Generic diagnosis.*—Heterophyidae: Body piriform in outline, strongly flattened dorso-ventrally. Oral sucker terminal; acetabulum well developed, lateral, directed antero-medially, and opening into a shallow genital sinus as in *Metagonimus*. Seminal vesicle slender and showing several constrictions. Testes oval, situated opposite each other at posterior end of body. Ovary irregular in shape, pretesticular, slightly to right of median line; seminal receptacle well developed; Laurer's canal present. Vitellaria lateral, extending from level of base of pharynx to posterior end of body. Uterus S-shaped, never extending caudad of ovary. Excretory vesicle Y-shaped, with short, wide stem and relatively short branches. Parasitic in intestinal tract of mammals.

*Type species.*—*Metagonimoides oregonensis* new species.

**Metagonimoides oregonensis** new species.

Figs. 1, 2.

*Description.*—*Metagonimoides:* Body piriform in outline, 589 to 688$\mu$ long by 573 to 852$\mu$ wide, strongly flattened dorso-ventrally. The cuticle is missing in all specimens available and it is not possible to determine whether spines are present or not. Oral sucker cup-shaped, 93 to 114$\mu$ in diameter, terminal

[1] Received August 3, 1931.

in position, and with its aperture slightly subterminal; prepharynx very short; pharynx strongly muscular, 77 to 109$\mu$ long by 46 to 93$\mu$ wide; esophagus short and slender; intestinal cæca slender and extending to posterior end of body. Acetabulum 124 to 233$\mu$ in diameter, situated in equatorial zone at right side of body, its aperture directed antero-medially and opening into a shallow genital sinus. Opening of genital sinus irregular and with two papilla-like ridges projecting into sinus, the larger of these ridges situated at postero-median border and the smaller situated at anterior margin. These structures have been interpreted by Witenberg[2] as a modified gonotyl or so-called genital

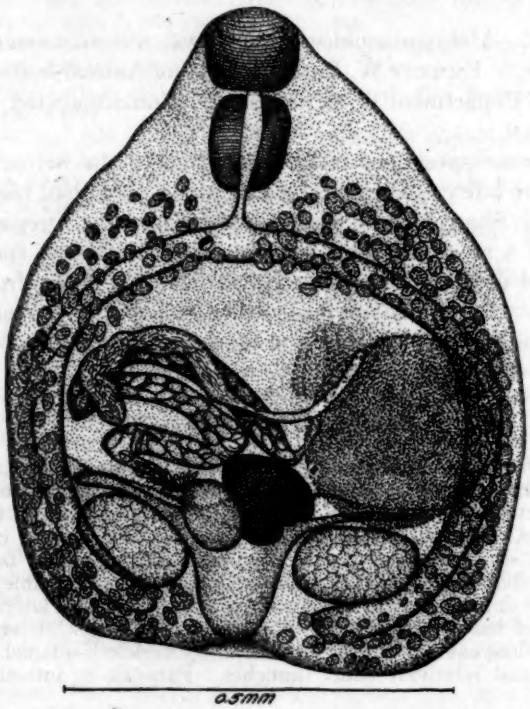


Fig. 1.—*Metagonimoides oregonensis* Price. Dorsal view.

sucker. No cirrus pouch present. Seminal vesicle slender, shaped somewhat like a transversely placed interrogation mark, and lying dorsal to uterus in left lateral field. Hemaphroditic duct opens into genital sinus at base of anterior ridge-like papilla. Testes oval, 112 to 150$\mu$ by 150 to 200$\mu$, and situated opposite each other in posterior part of body, their median border being separated by the stem of excretory vesicle. Ovary more or less irregular in outline, at posterior border of acetabulum in some specimens and median

[2] G. WITENBERG. *Studies on the trematode-family Heterophyidae.* Ann. Trop. M. and Parasit., Liverpool. **23**: 131-239, figs. 1-33. 1929.

to it in others; Laurer's canal present. Vitellaria lateral and composed of medium sized follicles which extend from level of posterior end of pharynx to posterior end of body; they are separated anteriorly and posteriorly by a narrow space. Uterus relatively short and forming a low S-shaped loop which lies chiefly in left lateral field; it does not extend caudad of ovary as in the case of *Metagonimus*. Eggs oval, 33$\mu$ long by 18$\mu$ wide, golden yellow in color. Excretory vesicle Y-shaped, with wide stem and relatively short, slender branches; excretory pore terminal.

*Host.*—*Procyon lotor*.

*Location.*—Small intestine.

*Distribution.*—United States (Corvallis, Oregon).

*Specimens.*—U. S. National Museum Helminthological Collection No. 30862 (type and paratypes).

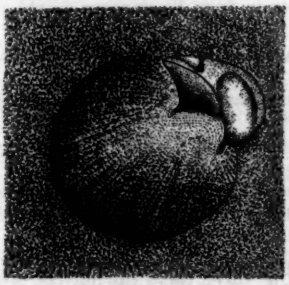

Fig. 2.—*Metagonimoides oregonensis* Price. Acetabulum and genital sinus. Ventral view.

This trematode is distinguished from species of *Metagonimus* (*M. yokogawai* (Katsurada), and *M. ciureanus* (Witenberg) ( = *Dexiogonimus ciureanus* Witenberg) by the distribution of the vitellaria and the course of the uterus. In *Metagonimus* the vitellaria do not extend anteriorly beyond the ovarial zone and the uterine coils extend caudad of the ovary and seminal receptacle, while in *Metagonimoides* the vitellaria extend as far anteriorly as the pharynx and the uterine coils do not extend caudad of the ovary and seminal receptacle.

## SCIENTIFIC NOTES AND NEWS

### Sixteenth International Geological Congress

According to a circular letter recently issued by the committee on organization of the International Geological Congress, the sixteenth session will meet in Washington in June, 1933. The following topics for discussion have been proposed:

Measurement of geologic time by any method.
Batholiths and related intrusives.
Zonal relations of metalliferous deposits.
Major divisions of the Paleozoic system.

Geomorphogenic processes in arid regions and their resulting forms and products.
Fossil man and contemporary faunas.
Orogenesis.

It is planned to offer the following excursions before the sessions:

A-1. An 11-day tour of eastern New York and western New England by bus, starting from New York City.

A-2. A 12-day tour of the mining districts of the southern Appalachians and the Mississippi Valley by special train from Washington.

A-3. A 10-day trip by bus through the Appalachian Valley in Virginia and portions of bordering States, from Washington.

A-4. A 10- or 11-day tour of northern New York by bus, from Albany.

A-5. A 10-day trip by boat to classic Cretaceous, Tertiary, and Pleistocene localities on Potomac River and Chesapeake Bay, from Washington.

A-6. An 11-day trip by train from Washington to the oil fields of Oklahoma and Texas.

A-7. A 5-day trip by bus from New York to Washington, primarily for geomorphologists.

A-8. A 4- or 5-day trip from New York to Washington by bus, primarily for economic geologists.

A-9. A transcontinental excursion, lasting 12 or 14 days, from San Francisco to Washington.

There will also be a series of 12 short excursions of one or two days length, starting from New York. Some of these will be offered before and others after the "A" series of excursions.

It is proposed to devote three days during the session to short excursions to points of interest near Washington.

The following excursions after the session are planned:

C-1. A transcontinental excursion by special train from Washington, 35 days. The major features of this excursion will be a series of side trips, generally by bus, to points of particular interest.

C-2. A transcontinental excursion by train from Washington, 35 days. This excursion has been planned so as to give a general cross section of the geology of the United States. The main emphasis is on regional and structural geology, physiography, and Tertiary stratigraphy, including volcanics.

C-3. A 10-day excursion by train to Chicago, thence by bus, to study the glacial geology of the Central States.

C-4. A 10- or 11-day excursion to the iron and copper deposits of the Lake Superior region.

C-5. A 35-day transcontinental excursion from New York or Washington to San Francisco by airplane with return by train, primarily for geomorphologists.

Copies of the circular, which contains detailed information regarding these excursions, may be obtained from the General Secretary, Sixteenth International Geological Congress, U. S. Geological Survey, Washington, D. C.